University of Bremen

– **Faculty 3** –

Courses in the
Winter Semester 2022/23

M.Sc. Industrial Mathematics & Data Analysis

M.Sc. Mathematics

M.Sc. Mathematik

M.Sc. Technomathematik

August 2022

This brochure summarizes the courses and lectures for the Master's Industrial Mathematics & Data Analysis, Mathematics, Mathematik (German-language), and Technomathematik (German-language) in the winter semester 2022/23. Further information can be found in the Course Catalog of the University of Bremen. There you will find, among other things, the language, the assignments to the individual modules, and the course code. The latter one can also be used to find a course in Stud.IP.

As you can see in the Course Catalog, all courses are in general assigned with an area of focus or specialization. For the M.Sc. Industrial Mathematics & Data Analysis, these are Data Analysis and Industrial Mathematics. For the M.Sc. Mathematics and the M.Sc. Mathematik, these are Algebra, Analysis, Numerical Analysis, and Statistics/Stochastics.

At this point we would like to refer to the Offers for International Students as well as to Living on Campus for answers regarding living, housing, financial help, and scholarships.

Contact

Academic Advisory Office - Mathematics

Place to go for questions on study programs, planning, recognition of credits and exam results, study abroad, and examination regulations. Also responsible for the design of this brochure.

Lars Siemer
Room / Number: MZH Building / 1300
+49 (0) 421 218 63744
szmathe@uni-bremen.de

www.szmathe.uni-bremen.de

Contents

Lectures

Algorithmic Game Theory 1

Algorithms and Uncertainty 3

Commutative Algebra 6

Introduction to Nonlinear Optimization
and Optimal Control 8

Inverse Problems 9

Mathematical Foundations of Data Analysis 11

Mathematical Foundations of Deep Learning 13

Numerical Methods for Partial Differential Equations 15

Sampling Theory and Methods 17

Symmetric Spaces 20

Theory of Nonparametric Tests 22

Seminars

Advanced Topics in Statistics 24

Dirichlet Forms on Graphs 26

High-Performance Visualization 27

Knots, the Universe and Everything 30

Mathematical Foundations of AI 32

Mathematical Modeling and Scientific Computing 33

Modellierungsseminar - Teil 2 35

Numerical Methods for Partial Differential Equations **37**

Trajectory Optimization **38**

The lecturers are responsible for the content of the descriptions

Algorithmic Game Theory

Course Code: 03-M-SP-6

Prof. Dr. Daniel Schmand, Elias Pitschmann

Contact: schmand@uni-bremen.de

Description

Many every-day processes can seen as a game between autonomous interacting players, where each player acts strategically in order to pursue her own objectives. This lecture is an introduction to game-theoretic concepts and techniques, mainly with connections to applications. Use-cases are distributed systems, auctions, online-markets, resource allocation, and traffic networks. The goal of the lecture is to provide an overview over state-of-the-art results in the area of algorithmic game theory. Main topics that we will cover in the course are

- Strategic Games and Efficiency of Equilibria
- Auctions, Truthfulness and VCG-mechanisms
- Cooperative Games
- Social Choice

Prerequisites

The course is designed for Master's students in Mathematics and Industrial Mathematics and Data Analysis. Bachelor's students in higher semesters and computer science students are also welcome.

Area of Focus or Specialization

M.Sc. Industrial Mathematics & Data Analysis:

- Industrial Mathematics

M.Sc. Mathematics and M.Sc. Mathematik:

- Numerical Analysis

Format and Examination

- Lectures: Thursdays and Fridays, 12.15-13.45, MZH 2340
- Exercise Sessions: Mondays, 14.15-15.45, MZH 4140
- Homework: weekly
- Exam: oral (if not too many participants)

Algorithms and Uncertainty

Course Code: 03-IMAT-AU

Prof. Dr. Nicole Megow, Dr. Felix Hommelsheim

Contact: nmegow@uni-bremen.de, fhommels@uni-bremen.de

Description

A key assumption of most powerful optimization methods is that all the input data is fully accessible at any time. However, from the point of view of many real-world applications (e.g., in logistics, production or project planning, cloud computing, etc.) this assumption is simply not true. Large data centers allocate resources to tasks without knowledge of exact execution times or energy requirements; transit times in road networks are often uncertain and depend on weather, traffic or accidents; in other applications, parameters such as bandwidth, customer demands or energy consumption are highly fluctuating. The current trend of data collection and data-driven applications often amplifies this phenomenon. As the amount of available data is increasing tremendously due to internet technology, cloud systems and sharing markets, modern algorithms are expected to be highly adaptive and learn and benefit from the dynamically changing mass of data.

The class "Algorithms and Uncertainty" will teach the most common frameworks of modeling uncertainty in the input data and discusses different methods on how to design and analyze efficient algorithms in these different models.

Specifically, we will cover the theory of **online optimization**, where initially unknown input data is revealed incrementally and needs to be processed immediately, before the next piece of information arrives. This model is best suited for analyzing critical networking and scheduling systems where devices and algorithms must perform well even in the worst-case scenario.

In the cases where previous history can be used to model the upcoming data, we often employ **robust optimization** or **stochastic optimization**. In robust optimization, the aim is to optimize the worst-case of all possible realizations of the input data. Hence, this model is rather conservative. In stochastic optimization however, the algorithms work with the assumption that data is drawn from some probability distribution known ahead of time and typically the goal is to optimize the expected value.

We also discuss modern, new lines of research going beyond the traditional models. Nowadays, another source of information is often available: machine learning algorithms can generate predictions which are accurate most of the time. However, there is no guarantee on the quality of the prediction, as the current instance may not be covered by the training set. This statement motivated a very recent research domain that will be covered in this course: how to use **error-prone predictions** in order to improve guaranteed algorithms.

Format and Examination

The course aims mostly at **Master's** students, although **Bachelor's students in higher semesters** are also very welcome. We teach the course with **4 hours per week** (**4 SWS** for **6 CP**), where roughly every other week one class will be an interactive exercise session.
There is the possibility to further extend the content of the course as well as the credits (**+3 CP**) by a seminar-style contribution (individual study of a recent research article and presentation to the rest of the class).

Examination: The examination will be by individual oral exam. As admission to the oral exam it is mandatory to present solutions in the exercise session at least twice during the term.

Prerequisites

Having heard an introductory course to discrete algorithms and their mathematical analysis (e.g. "Algorithmische Diskrete Mathematik", "Algorithmentheorie") is beneficial but it is not a requirement.

Area of Focus or Specialization

M.Sc. Industrial Mathematics & Data Analysis:

- Data Analysis
- Industrial Mathematics

M.Sc. Mathematics and M.Sc. Mathematik:

- Algebra
- Numerical Analysis

Literature

- Borodin and El-Yaniv: Online Computation and Competitive Analysis, Cambridge University Press, 1998
- A Ben-Tal, L El Ghaoui, A Nemirovski: Robust Optimization, Princeton University Press, 2009
- Most of the presented material is more recent and we will point to the corresponding research articles.

Commutative Algebra

Course Code: 03-M-SP-7

Prof. Dr. Anastasios Stefanou

Contact: stefanou@uni-bremen.de

Description

Commutative algebra is a branch of algebra which studies *commutative rings, ideals* and *modules*. *Algebraic geometry* and *algebraic number theory* build on commutative algebra. Commutative algebra is the main technical tool for the study of *varieties* and more generally, for the study of *schemes*. *Combinatorial commutative algebra* is utilized for the study of *multigraded rings* and *modules* and for their combinatorial presentations. These combinatorial presentations have been quite fundamental for the theory of *multiparameter persistent homology* in *topological data analysis*. In this course we will discuss the theory of *graded* and *multigraded modules*, *Gröbner bases*, *presentations* and *resolutions of modules*. Along the way, we will discuss interesting connections between multigraded modules and *multiparameter persistence* in topological data analysis. Topics include:

- Polynomial rings, ideals and varieties
- Gröbner bases and Buchberger's algorithm
- Hilbert's Nullstellensatz
- Multigraded modules and Betti numbers
- Rank invariants
- Presentations and resolutions
- Hilbert's syzygy theorem
- Schreyer's theorem
- Flat-injective presentations

Prerequisites

The prerequisites are Linear Algebra 2 – or – Mathematics for Computer Science

Times and Formalities

The course is a 4+2 lecture, i.e. there are weekly two 2-hour lectures and a 2-hour exercise session. The academic performance consists of successfully completing (at least 50% of the points of) the HW exercises and then passing the module test. The module test will consist of a set of problems that must be returned to the lecturer no more than a week later. The grade of the course will be the grade of the module test.

Area of Focus or Specialization

M.Sc. Mathematics and M.Sc. Mathematik:

- Algebra

Literature

[1] D. A. Cox, J. Little, D. O'Shea. *Ideals, varieties and algorithms*, Springer.

[2] D. S. Dummit, R. M. Foote, *Abstract Algebra*, Wiley.

[3] D. A. Cox, J. Little, D. O'Shea, *Using algebraic geometry*, Springer.

[4] E. Miller, B. Sturmfells, *Combinatorial commutative algebra*, Springer.

[5] G. Carlsson, G. Singh, A. J. Zomorodian, *Computing multidimensional persistence* , Journal of Computational Geometry, 1.1: 72–100, 2010.

[6] E. Miller, *Homological algebra for modules over posets*, https://arxiv.org/abs/2008.00063.

[7] A. L. Thomas, *Invariants and Metrics for Multiparameter Persistent Homology.* PhD Thesis. Duke University, 2019.

[8] W. Kim, F. Mémoli, *Generalized persistence diagrams for persistence modules over posets.* Journal of Applied and Computational Topology, 2021, 5.4: 533-581.

Introduction to Nonlinear Optimization and Optimal Control

Course Code: 03-M-SP-9

Prof. Dr. Christof Büskens

Contact: bueskens@uni-bremen.de

Description

The famous mathematician Leonhard Euler once said "Nothing takes place in the world without some maximum or minimum principle arises." or in other words optimization takes place everywhere. Electricity takes the path of lowest impedance, light takes the path of shortest time, physical systems tend to a state of minimum energy, ...
And man also optimizes (or at least tries to): Train journeys are optimally coordinated, workers are allocated in the best possible way, traffic flows are maximized, companies maximize their profit and insurers try to minimize damage.
In mathematics, optimization is a key technology to connect science with application. Uniquely solvable systems of equations are even rather a special case, because normally systems are overdetermined and we search for e.g. digital twins by means of optimization methods or they are underdetermined and one would like to use the remaining degrees of freedom to optimize a superior performance measure.
In this lecture we will cover some of the basic principles of optimization in the static as well in the dynamic case.

Area of Focus or Specialization

M.Sc. Industrial Mathematics & Data Analysis:

- Industrial Mathematics

M.Sc. Mathematics and M.Sc. Mathematik:

- Numerical Analysis

Inverse Problems

Course Code: 03-M-SP-1

Dr. Matthias Beckmann, Prof. Dr. Peter Maaß

Contact: matthias.beckmann@uni-bremen.de

Description

This course gives a mathematical introduction to the classical theory of linear inverse problems, which occur in many scientific fields like physics, engineering and biology. Important areas of application include optics, radar, acoustics, communication theory, signal processing, medical imaging, computer vision, geophysics, oceanography, astronomy and non-destructive testing.
Loosely speaking, solving the socalled forward problem consists of computing the outcome of a known model given the model parameters. Conversely, the inverse problem consists of determining unknown parameters of interest given the model and noisy measurements of the outcome. It is called an inverse problem because it starts with the effects and then calculates the causes. In this sense it is the inverse of the forward problem, which starts with the causes and then calculates the effects.
Solving an inverse problem suffers from additional mathematical difficulties. First of all, the observations do typically not contain enough information and additional knowledge is required. Second, the unavoidable noise may cause the measurements to not being the image of any model so that we cannot look for model parameters that produce the data exactly.
In this course we will discuss different approaches to tackling these difficulties and deal with various concrete methods for solving linear inverse problems. In particular, the following topics will be addressed:

- Examples of inverse problems
- Notion of ill-posedness
- Regularization methods and optimality
- Truncated singular value decomposition
- Tikhonov regularization
- Landweber iteration

Concrete examples of inverse problems studied in this course are image reconstruction problems in medical imaging such as computerized tomography, deconvolution problems and denoising of signals like images.

Prerequisites

Basics from B.Sc. courses in Mathematics (calculus, linear algebra, numerics) and basic programming skills. Prior knowledge in functional analysis is advantageous, but fundamental theorems will be stated in the course.

Area of Focus or Specialization

M.Sc. Industrial Mathematics & Data Analysis:

- Data Analysis
- Industrial Mathematics

M.Sc. Mathematics and M.Sc. Mathematik:

- Analysis
- Numerical Analysis

Times and Formalities

The course, comprising 4+2 hours per week, is split into a lecture series (4h each week), presumably

Wed. 14-16 in MZH 2340 & Fr. 10-12 in MZH 2340

and accompanying exercise classes (2h each week), presumably

Fr. 8-10 in MZH 2340.

It is necessary to solve the provided exercise sheets and actively participate in the exercise classes. Exercises will be assigned every week and the students are requested to present their solutions during the exercise classes.

Literature

- H. Engl, M. Hanke, A. Neubauer, *Regularization of Inverse Problems*, Kluwer Academic Publisher, 2000.
- P.C. Hansen, *Discrete Inverse Problems*, SIAM, 2010.
- J. Kaipio, E. Somersalo, *Statistical and Computational Inverse Problems*, Springer, 2005.
- J. Mueller, S. Siltanen, *Linear and Nonlinear Inverse Problems with Practical Applications*, SIAM, 2012.
- A. Rieder, *Keine Probleme mit Inversen Problemen*, Vieweg, 2003.

Mathematical Foundations of Data Analysis

Course Code: 03-MDAIP-1

Prof. Dr. Peter Maaß

Contact: pmaass@uni-bremen.de

Description

This module gives an introduction into the mathematical foundations of data analysis. We start by a description of different data types (signals, images, multi-spectral data etc.) and discuss continuous a well as discrete data models. Motivated by real life problems, which require tasks such as denoising, contrast enhancement, edge detection, compression, clustering, classification or segmentation, we develop respective algorithms as well as their underlying theory. On the mathematical side this involves the modelling of data analysis tasks in function spaces, data transformations such as Fourier transforms and elements from partial differential equations.
This course will be continued in the semester Summer 2023 by a course on 'Mathematical Foundations of machine learning'. However, we will already include parts of machine learning in the present course on 'Mathematical foundations of data analysis'. In particular we will cover neural networks and deep learning in the second part of the present semester. Hence, starting around December 2022, this course will be merged with the course on 'Mathematical Foundations of Deep Learning', which is taught by Dr. Sören Dittmer.

Area of Focus or Specialization

As part of the module "Mathematical Methods for Data Analysis and Image Processing", mandatory for M.Sc. Industrial Mathematics & Data Analysis

M.Sc. Mathematics and M.Sc. Mathematik:

- Numerical Analysis

Times and Formalities

This course is organized in weekly lectures (2 lectures a 2h each) and exercise classes (2h per week). The students are required to submit their solutions to weekly exercise sheets. These exercises will require substantial theoretical work as well as implementation of algorithms. Completion of sufficiently many exercises as well as active participation in the exercise classes are are a prerequisite for the module exam at the end of the term. As already stated, this lecture is combined with the course on deep learing in the second half of the semester.

Literature

- K. Bredies und D. Lorenz, *Mathematische Bildverarbeitung: Einführung in Grundlagen und moderne Theorie*, Springer, 2011.
- R. Gonzalez und R. Woods, *Digital Image Processing*, Prentice Hall, 4. Ausgabe, 2018

Mathematical Foundations of Deep Learning

Course Code: 03-M-MDAIP-2

Dr. Sören Dittmer

Contact: sdittmer@math.uni-bremen.de

Description

This course gives an introduction to the mathematical foundations of deep learning. It will start around December 2022 as a continuation of the "Mathematical Foundations of Data Analysis" course in the first half of the semester.

The course starts with the basics of neural networks, e.g., multi-layer perceptrons, convolutional layers, and gradient descent. On the one hand, we will examine these basics from a theoretical point of view. On the other hand, we will discuss how to implement them in PyTorch and run them on different types of data. Motivated by problems and approaches introduced in the course "Mathematical Foundations of Data Analysis," we will explore how deep learning can augment and improve upon classical results.

The second part of the course will dive into more advanced topics in deep learning: E.g., unsupervised learning, generative modeling, recent theoretical advancements, and open theoretical problems.

Area of Focus or Specialization

As part of the module "Mathematical Methods for Data Analysis and Image Processing", mandatory for M.Sc. Industrial Mathematics & Data Analysis

M.Sc. Mathematics and M.Sc. Mathematik:

- Numerical Analysis

Times and Formalities

This course involves two weekly lectures (each 2h) and one weekly exercise class (2h). The students are required to submit their solutions to weekly exercise sheets. These exercises will require substantial theoretical work as well as the implementation of algorithms. Prerequisites for the module's exam at the end of the term are the completion of sufficiently many exercises and active participation in the exercise classes. As already stated, this lecture is combined with the data analysis course in the semester's first half.

Literature

- Goodfellow, Ian, Yoshua Bengio, and Aaron Courville. *Deep learning*. MIT press, 2016.

- Paszke, Adam, et al. *Automatic differentiation in pytorch.* (2017).

Numerical Methods for Partial Differential Equations

Course Code: 03-M-NPDE-1

Prof. Dr. Andreas Rademacher

Contact: arademac@uni-bremen.de

Description

Partial differential equations are a main component in the modelling of physical, chemical or biological phenomena in several spatial dimensions or in space and time. They also often occur in mathematical problems in geometry or calculus of variations. The lecture deals with the discretisation of partial differential equations and the estimation of the error between continuous and discrete solution. In particular, the finite element method is introduced and investigated, with special consideration of modern adaptive algorithms. We will first discuss the application of the method to stationary elliptic problems. Later also time-dependent problems will be considered. The connection of theory, numerical analysis and implementation is particularly important. Facts from the theory of partial differential equations are usually only quoted. Based on special application-oriented chapters of the lecture, the numerical algorithms are to be implemented in programming tasks under guidance.

Prerequisites

Good knowledge in analysis, linear algebra, and numerics. Knowledge of functional analysis is advantageous. Programming knowledge in Matlab for the practical exercises.

Area of Focus or Specialization

Mandatory for M.Sc. Industrial Mathematics & Data Analysis as well as for M.Sc. Technomathematik

M.Sc. Mathematics and M.Sc. Mathematik:

- Numerical Analysis

Times and Formalities

There are two lectures and one exercise per week. Every week an exercise sheet is published with theoretical and, if applicable, practical tasks. For the award of the course credit, 50% of the possible points must be achieved in the exercises. Following the lecture, oral examinations take place on dates to be agreed upon.

Literature

An overview of various methods for partial differential equations is provided, for example, by the following books:

- Braess, D.; Schumaker, L. L.: Finite elements - theory, fast solvers, and applications in elasticity theory, Cambridge University Press, Cambridge, third edition, 2010

- Knabner, P.; Angermann, L.: Numerical Methods for Elliptic and Parabolic Partial Differential Equations, Springer, Cham, second edition, 2021

- Johnson, C.: Numerical solution of partial differential equations by the finite element method, University Press, Cambridge, 1987

Sampling Theory and Methods

Course Code: 03-M-SP-8

Dr. Maryam Movahedifar

Contact: movahedm@uni-bremen.de

Description

Sampling theory is the field of statistics that is involved with the collection, analysis, and interpretation of data gathered from random samples of a population under study. The application of sampling theory is concerned not only with the proper selection of observations from the population that will constitute the random sample; it also involves the use of probability theory, along with prior knowledge about the population parameters, to analyze the data from the random sample and develop conclusions from the analysis. The normal distribution, along with related probability distributions, is most heavily utilized in developing the theoretical background for sampling theory. So, in order to answer the research questions, it is doubtful that the researcher should be able to collect data from all cases. Thus, there is a need to select a sample. The entire set of cases from which the researcher's sample is drawn is called the population. Since researchers neither have the time nor the resources to analyze the entire population so they apply the sampling technique to reduce the number of cases. Furthermore, as there are different types of sampling techniques/methods, the researcher needs to understand the differences to select the proper sampling method for the research. In this regard, this course also presents the different types of sampling techniques and methods. In general, sampling techniques can be divided into two types:

- Probability or random sampling
- Non- probability or non- random sampling

Before choosing a specific type of sampling technique, it is needed to decide broad sampling technique. Figure 1 shows the various types of sampling techniques.

The course is organized as follows: Section 1 will be introduced the basic concepts of Sampling Theory which are essential to understanding the later sections. Some numerical examples are also will be presented to help have a clear understanding of the concepts. A simple random sampling design will be dealt with in detail in the second section. Several solved examples which consider various competing estimators for the population total will

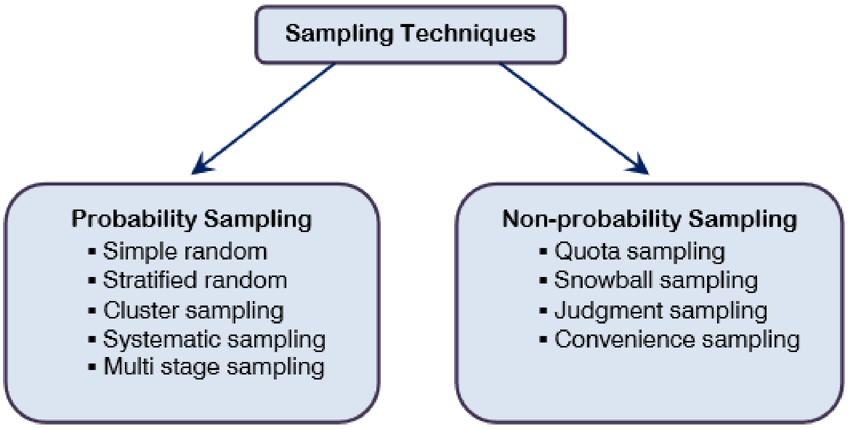


Figure 1: Sampling Techniques.

also be included in the same section. The third is devoted to systematic sampling schemes. Various systematic sampling schemes like linear, circular, and balanced. Modified systematic sampling and their performances under different super population models will also be discussed. In the fourth section, several unequal probability sampling-estimating strategies will be presented. Probability Proportional to Size Sampling With and Without Replacement will be considered with appropriate estimators. Stratified sampling, allocation problems, and related issues will be presented with full details in the fifth section. Many interesting solved problems are also will be added. In the sixth and seventh sections, the use of auxiliary information in ratio and regression estimation will be discussed. Results related to the properties of ratio and regression estimators under super-population models are also will be given. Cluster sampling and Multistage sampling will be presented in the eighth section.

Prerequisites

- No formal requirements, but knowledge of proper backgrounds of statistical concepts and methods is strongly recommended.

Area of Focus or Specialization

M.Sc. Mathematics and M.Sc. Mathematik:

- Statistics / Stochastics

Times and Formalities

This course will be presented as lectures plus exercises = 9 CP as an in-person class.
Regular participation in the course, working on classroom tasks, working on 75% of the homework in a meaningful way and a final written exam, will determine the final score.

Literature

- S. Sampatb (2001). Sampling Theory and Methods. CRC Press Boca Raton London.
- Pascal Ardilly, Yves Tillé (2006). Sampling Theory and Methods. Springer-Verlag GmbH
- Luca Martino, David Luengo, Joaquín Míguez ,(2018). Independent Random Sampling Methods. Springer-Verlag GmbH

Symmetric Spaces

Course Code: 03-M-SP-4

Prof. Dr. Anke Pohl

Contact: apohl@uni-bremen.de

Description

A Riemannian symmetric space is a connected Riemannian manifold which admits an isometric geodesic symmetry (a "time reversal") about every point. Riemannian symmetric spaces are fundamental objects in a variety of situations in both mathematics and physics. The existence of these isometric geodesic symmetries has the effect that the Riemannian isometry group carries a natural structure of a finite-dimensional manifold and hence is a so-called Lie group. Essentially all properties of a Riemannian symmetric space can be studied via its associated Lie group. However, the story goes further. In a natural way, the tangent space of a Lie group at the identity element carries the structure of an algebra with a binary algebra operation (a product within the algebra) that obeys the so-called Jacobi identity. In other words, it is a Lie algebra. Again, the situation enjoys sufficient rigidity that essentially all properties of the Lie group, and hence of the Riemannian symmetric space, can be studied via properties of the associated Lie algebra. The interplay between Riemannian symmetric spaces, Lie groups and Lie algebras is a nature's masterpiece of elegant and efficient mathematics and, in particular, of synergy of analysis and algebra. In this course we will study and prove this interplay after a rigorous and gentle introduction of all necessary objects. This course may well serve as a basis for a subsequent master's thesis, a reading course or further, advanced courses in this research area.

Prerequisites

Well-founded mathematical knowledge to the extent of a bachelor's degree in mathematics. No prior knowledge of differential geometry, manifold theory or Lie theory is expected.

Area of Focus or Specialization

M.Sc. Mathematics and M.Sc. Mathematik:

- Algebra

- Analysis

Formalities

This course takes place as a "small" course, i.e.with 2 hours lecture (2 SWS lecture) and 1 hour tutorial/discussion (1 SWS tutorial) and is therefore a 4.5 CP course. The course on symmetric spaces can be combined with *any* other 4.5 CP course of a suitable specialization to a 9 CP course. Examinations are oral. The course credit consists of active participation in the exercise program.

Further Information

Further and more detailed information about examinations and course credit regulations will be communicated via Stud.IP at the beginning of the lecture period.

Theory of Nonparametric Tests

(Statistics III)

Course Code: 03-M-SP-5

Prof. Dr. Thorsten Dickhaus

Contact: dickhaus@uni-bremen.de

Description

Introductory statistics courses typically mainly deal with *parametric* statistical models and methods. In particular, likelihood-based inferential theory is covered in "Statistics I" and "Statistics II". However, parametric model assumptions are often hard to check or to justify in practice. In the course "Statistics III (Theory of Nonparametric Tests)", we therefore investigate *nonparametric* statistical models and derive statistical tests under such models. Likelihood-based methods will in this context be replaced by the *substitution principle* or, synonymously, the plug-in method: The unknown probability distribution of the (random) data is replaced by their empirical distribution. For a given null hypothesis H_0, the "discrepancy" between the empirical distribution and the model subspace corresponding to H_0 is quantified and tested for significance.

The specific topics of the course are:

- Empirical measure, empirical cumulative distribution function, empirical processes
- Goodness-of-fit tests
- Rank tests
- Resampling tests
- Projection tests, e. g., "empirical likelihood" tests

Area of Focus or Specialization

M.Sc. Industrial Mathematics & Data Analysis:

- Data Analysis
- Industrial Mathematics

M.Sc. Mathematics and M.Sc. Mathematik:

- Statistics/Stochastics

Times and Formalities

Upon successful completion of the course, 9 ECTS points will be granted. The course consists of 4 hours lecture plus 2 hours exercise class per week. Exercise sheets will be issued every week and students' solutions will be corrected. This serves the purposes of (i) monitoring the learning progress and (ii) gaining the study credits. The final examinations will be oral exams.

Literature

- Dickhaus, T. (2018): Theory of Nonparametric Tests. Cham: Springer, ISBN 978-3-319-76314-9.

- Edgington, E.S., Onghena, P. (2007): Randomization tests. With CD-ROM, 4th Edition. Boca Raton, FL: Chapman & Hall/CRC, ISBN 978-0367577711.

- Efron, B., Tibshirani, R.J. (1993): An introduction to the bootstrap. Monographs on statistics and applied probability, Vol 57. New York, NY: Chapman & Hall, ISBN 978-0412042317.

- Hájek, J., Šidák, Z., Sen, P.K. (1999): Theory of rank tests, 2nd Edition. Orlando, FL: Elsevier/Academic Press, ISBN 0-12-642350-4.

- Owen, A.B. (2001): Empirical likelihood. Boca Raton, FL: Chapman & Hall/CRC, ISBN 1-58488-071-6.

Advanced Topics in Statistics

Course Code: 03-M-AC-7

Prof. Dr. Vanessa Didelez

Contact: vdidelez@uni-bremen.de

Description

This course is a 'seminar' which means that the students will prepare the topics and give presentations. We will consider a number of different (advanced) topics of which statisticians should be aware, but that are often not addressed in basic statistics lectures. These include for instance:
- statistical methods for measurement error
- statistical methods for missing values
- statistical methods for high-dimensional data
- conditional independence and graphical models
- causal inference
- sensitivity and bias analyses
- machine learning methods
- statistical method for the analysis of networks
- mixed / random effects models for longitudinal data

Students may also suggest their own topics.

The participants will receive a list of relevant literature (or can find literature and sources themselves).

We will also discuss what makes a good presentation and written report; this should help students to prepare for their master's thesis.

Prerequisites

The seminar is aimed at master-level students of mathematics with focus on statistics (it can be opened up to others – please let me know if you are interested). Prerequisites are a basic knowledge of statistics / stochastics.

Area of Focus or Specialization

M.Sc. Industrial Mathematics & Data Analysis:

- Data Analysis

M.Sc. Mathematics and M.Sc. Mathematik:

- Statistics/Stochastics

Times and Formalities

A preparatory meeting will take place on Tuesday 18 October 4:15pm. If you are interested but cannot make the time/date, please let me know (from experience there is always some flexibility and we will try to accommodate everyone).

Also, if you are willing to present early during the term (e.g. November) and would like to start reading-up before term starts, please let me know.

The seminar consists of 90min per weeks, where we meet, and some preparation / reading in your own time; the topics will be distributed among the participants who will then prepare a 60min presentation at a level that all other students can understand, followed by questions and answers; in addition, students will prepare a ca. 20-page written report on their topic (the length will be adapted to levels / credit points). The final grade is made up of the quality of the oral presentation and written report.

Literature

The following books contain relevant chapters to various topics (but this is not an exhaustive list):

- Carpenter & Kenward (2012): *Multiple imputation and its applications.*
- Carroll, Ruppert, Stefanski, & Crainiceanu, (2006): *Measurement error in nonlinear models: a modern perspective.*
- Hastie, Tibshirani & Friedman (2009): *Elements of Statistical Learning.*
- Kolaczyk (2009): *Statistical Analysis of Network Data: Methods and Models.*
- Lauritzen (1996): *Graphical Models.*
- Pearl (2009): *Causality: Models, Reasoning, Inference.*

Dirichlet Forms on Graphs

Course Code: 03-M-AC-10

Dr. Hendrik Vogt

Contact: hendrik.vogt@uni-bremen.de

Description

The seminar is based on the so-called “Internet Seminar”, see

https://www.mat.tuhh.de/veranstaltungen/isem26/

On that website, new material will be provided each week, starting mid October. This material shall be presented by the participants.

The seminar is devoted to the treatment of graphs and discrete Dirichlet spaces. We will investigate discrete Laplacians associated with Dirichlet forms on graphs. Here, a *graph* is a discrete and countable set X of vertices together with a measure m on X of full support and an edge weight $b\colon X \times X \to [0, \infty)$ satisfying suitable assumptions. (Then $b(x, y) > 0$ will be interpreted as there being an edge between the vertices $x, y \in X$.)

A continuation in the summer semester is possible: there will be a project phase where you can work in small international groups of 3–4 students and a workshop in Wuppertal (July 16 to July 22, 2023) where participants meet together with leading experts in the field.

Prerequisites

Basic knowledge in functional analysis is highly recommended (bounded operators, uniform boundedness principle, closed graph theorem, Hahn-Banach theorem, foundations of Hilbert spaces), as well as on foundations in complex analysis.

Area of Focus or Specialization

M.Sc. Mathematics and M.Sc. Mathematik:

- Analysis

Times and Formalities

Apart from registering on Stud.IP, please also register via the website above to get access to the seminar material.

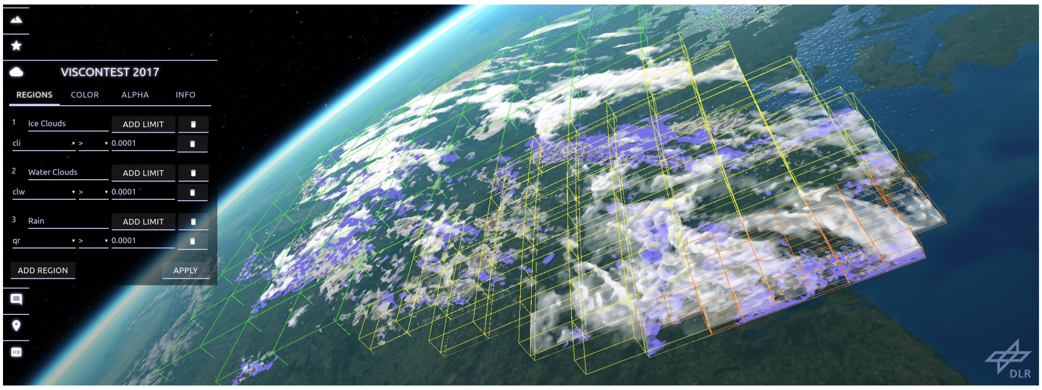
High-Performance Visualization

Selected publications from the field of
visualization of large scientific datasets

Course Code: 03-M-AC-2

Prof. Dr. Andreas Gerndt

Contact: gerndt@uni-bremen.de



Description

The seminar deals with the mathematical basics of scientific visualization and covers methods for parallel post-processing of large scientific datasets. A wide variety of scientific applications make use of such data. On the one hand, they are generated by simulations on high-performance computers (e.g., to support climate research or to predict airflow around aircraft wings). But they can also be generated by measurements e.g. by Earth observation missions. In order to obtain meaningful information for visualization, these enormously large raw data must first be processed. For a subsequent explorative analysis, real-time interactive methods are needed, which in turn rely on highly parallel and efficient approaches. The seminar therefore addresses current trends in scientific visualization. Outstanding publications by leading scientists will be selected, covering topics ranging from multi-resolution extraction of topology features to parallel acceleration methods for volume rendering in virtual working environments.

Prerequisites

The seminar is open to students from the fields of mathematics, computer science or from other relevant domains (such as geosciences or aerodynamics). Previous participation in the lecture "High-Performance Visualization" would be helpful. However, this is not a prerequisite. Knowledge in Computer Graphics or High-Performance Computing (HPC) might also be helpful.

Area of Focus or Specialization

M.Sc. Industrial Mathematics & Data Analysis:

- Data Analysis

M.Sc. Mathematics and M.Sc. Mathematik:

- Numerical Analysis

Times and Formalities

In the introductory session, selected publications from the field of scientific visualization will be presented. The students can then choose one of the papers. By Christmas, the underlying basic literature is to be researched. In the further course of the semester, the topic will then be prepared and a seminar paper of approximately 20 pages will be written. At the end of the semester, the papers will be presented. Depending on the number of participants, the presentations will take place in individual or block sessions. Students are expected to present the elaborated topics in 45-minute talks. The presentation and paper should preferably be in English. Seminar paper and presentation will be used for a performance evaluation. For questions and support, contact persons for the respective topics are available throughout the semester. The supervision will take place mainly online.

Literature

- A. C. Telea, "Data Visualization – Principles and Practice", 2. Edition, CRC Press, 2015
- E. W. Bethel, H. Childs, C. Hansen, "High Performance Visualization", CRC Press, 2013
- W. Schroeder, K. Martin, B. Lorensen, "The Visualization Toolkit", 4. Edition, Kitware, 2006

- C. Hansen, C. Johnson, "The Visualization Handbook", Elsevier Academic Press, 2005

Knots, the Universe and Everything

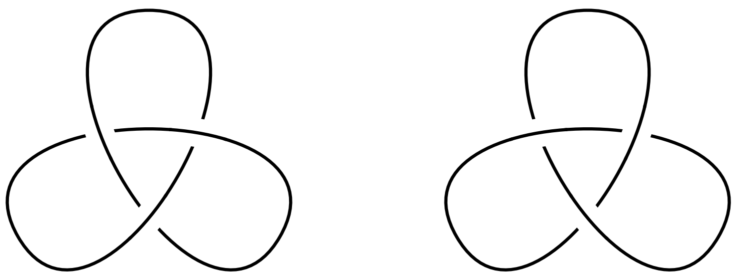
Course Code: 03-M-AC-9

Dr. Tim Haga

Contact: timhaga@uni-bremen.de

Description

An embedding of the circle S^1 into the Euclidean space $\mathbb{R}^3$ is called a knot. Of course, we can twist and stretch the knot, but we are not allowed to cut it. A main aspect of knot theory is to classify knots up to twisting and stretching. That means we are in search of an invariants for knots that do not change if we twist or stretch the knot.
If we have a proper knot in $\mathbb{R}^3$, we can not untangle it to a simple cirle. This statement is false for higher dimensions, where every knot can be unravelled. This fact might very well be the reason, why we live in a 3-dimensional world.



Two knots. Are they equivalent?

Knot theory is an active part of mathematics, with many connections to topics inside and outside of mathematics, such as topology, biochemistry, quantum physics, and more. In this seminar, we will study the basic concepts of this topic and some of the applications in other fields.

Organisation

The first seminar session takes place on Tuesday, October, 18th at 2 p.m. All following sessions will take place upon individual appointments.
In a first session we will discuss and distribute the topics. Each student is then asked to prepare a written elaboration on his or her topic and to present it in a talk. The seminar can be given either in English or German depending on the needs of the participants.

Prerequisites

A good understandig of linear algebra and algebra will suffice. A basic knowledge of topology might be helpful but will not be essential. We will cover the necessary parts of (algebraic) topology as we make our way through the subject.

Area of Focus or Specialization

M.Sc. Mathematics and M.Sc. Mathematik:

- Algebra

Literature

- Adams: The Knot Book, AMS 2004
- Burde, Zieschang, Heusener: Knots, De Gruyter 2014
- Cromwell: Knots and Links, Cambridge University Press 2004
- Flapan: Knots, Molecules and the Universe, AMS 2015
- Lickorish: An introduction to Knot Theory, Graduate Texts in Mathematics, Springer, 1997

Mathematical Foundations of AI

Course Code: 03-M-AC-6

Dr. Sören Dittmer

Contact: sdittmer@math.uni-bremen.de

Description

In this seminar, every participant has to read and understand a handful of papers about a mathematical topic in artificial intelligence, e.g., a subfield of deep learning. Participants can choose between applied but mathematical methods or theoretical foundations. In both cases, the participants should connect theory with numerics.

Prerequisites

- A bachelor's degree in (industrial) mathematics.
- Experience with the programming language python.

Area of Focus or Specialization

M.Sc. Industrial Mathematics & Data Analysis:

- Data Analysis
- Industrial Mathematics

M.Sc. Mathematics and M.Sc. Mathematik:

- Numerical Analysis

Formalities

The seminar requires a write-up of the reading material and the numerical results. It also requires a presentation about the topic.

Mathematical Modelling and Scientific Computing

Course Code: 03-M-AC-4

Prof. Dr. Andreas Rademacher

Contact: arademac@uni-bremen.de

Description

The seminar will deepen individual questions on the finite element method. Possible topics come from the following areas:

- Efficient solution of large systems of linear equations
- Mixed finite element methods
- Adaptive finite element methods
- Discretisation and solution of contact problems or other nonsmooth problems from structural mechanics, e.g. elasto-plasticity
- Optimal control of partial differential equations

Prerequisites

The course is based on the lecture numerical methods for partial differential equations.

Area of Focus or Specialization

M.Sc. Industrial Mathematics & Data Analysis:

- Data Analysis
- Industrial Mathematics

M.Sc. Mathematics and M.Sc. Mathematik:

- Numerical Analysis

Times and Formalities

The presentations are each based on an original paper. Based on a written paper, the topic will be presented in a lecture lasting 70-80 minutes. Please register for the seminar by email with the lecturer. There will be a preliminary meeting at the beginning of the lecture period, where the topics will be distributed and the further procedure will be coordinated.

Literature

The literature is specified when the seminar topics are assigned. This is original literature.

Modellierungsseminar - Teil 2

Course Code: 03-M-MS-2

Dr. Tobias Kluth

Contact: tkluth@math.uni-bremen.de

Veranstaltungsbeschreibung

Über zwei Semester hinweg bearbeiten die Teilnehmer*innen des Modellierungsseminars in Teamarbeit ein Projekt, bei dem sie ihre bereits erworbenen mathematischen Kenntnisse in Anwendungen außerhalb der Mathematik zum Einsatz bringen sollen. Die Projektpartner können dabei Industrieunternehmen oder Forschungsinstitute sein. Das Themenspektrum wird durch das Angebot der Projektpartner bestimmt. In diesem Jahr freuen wir uns auf eine Zusammenarbeit mit diesen Partnern (in alphabetischer Reihenfolge):

- Aidea (Bremen)
- Airbus (Bremen)
- ArcelorMittal (Bremen)
- Deutsches Zentrum für Luft- und Raumfahrt (DLR Bremen)
- Institut für Windenergietechnik, IWES (Bremerhaven)

Ablauf, Format und Prüfungsform

- Regelmäßige Meetings (Präsenz oder ggf. Online)
- Vorträge der Teilnehmenden über ihre Themen und aktutellen Bearbeitungsstand

Die Bewertung erfolgt zum Ende des Modellierungsseminars Teil 2 (im Februar 2023) anhand dieser Abgaben:

- interner mathematischer Vortrag und öffentlicher anwenderorientierter Vortrag
- schriftliche Ausarbeitung (ca. 30 Seiten)
- Poster (oder vergleichbares Format, z.B. Video, interaktive Software, Demonstrator)

Voraussetzungen

Dieses Seminar richtet sich an Studierende im Masterstudiengang Technomathematik. In begründeten Ausnahmefällen können auch Studierende im Masterstudiengang Mathematik teilnehmen. Voraussetzung für die Teilnahme an Teil 2 ist die Teilnahme an Teil 1.

Numerical Methods for Partial Differential Equations

Course Code: 03-M-AC-1

Prof. Dr. Alfred Schmidt

Contact: alfred.schmidt@uni-bremen.de

Description

Many models from applications lead to nonlinear partial differential equations or systems of partial differential equations. Their solution as well as aspects of optimization typically demand for numerical methods.
In the winter term 2022/23 we will especially look at numerical methods for models in engineering applications from milling and grinding processes. Numerical aspects of thermo-mechanics or thermo-fluiddynamics, as well as multi-scale-models or week formulations of boundary conditions may be subject of seminar talks.

Prerequisites

Advantageous will be a good knowledge of analysis and numerical methods, as well as some proficiency in numerical methods for partial differential equations, which is typically taught in the lectures of the same name.

Area of Focus or Specialization

M.Sc. Industrial Mathematics & Data Analysis:

- Industrial Mathematics

M.Sc. Mathematics and M.Sc. Mathematik:

- Numerical Analysis

Times and Formalities

Successful conclusion of the seminar demands an oral presentation of about 60 minutes plus discussion and a written elaboration about the subject of the talk.
A preliminary discussion of available subjects will be offered in the first semester week, but subjects can be assigned later, too. Just get in touch with the organizer.

Trajectory Optimization

Course Code: 04-M30-CP-SFT-3

Dr. Matthias Knauer

Contact: knauer@uni-bremen.de

Description

A look to the stars has always made people want to understand how the mechanics of the sky can be described. In the lecture we will examine some of the mathematical concepts that have developed over the centuries from this thirst for knowledge and which come from both pure and applied mathematics. To describe the undisturbed orbit of a planet, we need the solution of the Kepler equation, for which we get to know a variety of solution methods. With the change from the two-body problem to the three-body problem, the dynamic system becomes too complex for analytical solutions. Is it still possible to calculate something analytical under suitable restrictions? The orbits of the planets can be described by orbit elements. We investigate how the orbital elements change due to additional forces.
With the first steps into space, the view of earth also changed. A precise description of the earth was needed to keep satellites in orbit. We will discuss the mathematics behind it in the further course of the lecture in order not to exceed a manageable computational effort in the application. We first devote ourselves to the mathematical representation of gravitational fields and transfer these results to the earth.
When it comes to successful missions to other celestial bodies, the engineering power is unquestionable. But how efficient are these missions actually? Based on some typical orbital manoeuvres, landing on the moon and Mars or rendezvous maneuvers in earth orbit, we see what requirements are placed on missions.
With optimization and optimal control, mathematics offers tools to plan missions that are perhaps more resource-efficient or safer. In the lecture, we use the software WORHP Lab to present the terms and processes of these disciplines in a compact form. For real-time optimization, we apply parametric sensitivity analysis and learn about the concepts of model predictive control (MPC). We end the lecture with an outlook on the peculiarities of an on-board optimization.
The lecture is also offered to students of the Master's program in Space Engineering.

Prerequisites

- Numerical Analysis

Area of Focus or Specialization

M.Sc. Industrial Mathematics & Data Analysis:

- Industrial Mathematics

M.Sc. Mathematics and M.Sc. Mathematik:

- Numerical Analysis

Times and Formalities

If the circumstances allow it, this lecture will take place in presence.
The weekly exercises also include programming exercises (e.g. with MATLAB or WORHP Lab). The exercises are submitted and corrected digitally. All participants should present their solution to an exercise during the semester. There will be a written exam in March 2023 (and a repetition in August 2023).
The lecture consists of 2+1 SWS.

Literature

- J. Danby. Fundamentals of Celestial Mechanics. Willmann-Bell, 2003.
- D. Vallado. Fundamentals of Astrodynamics and Applications. Springer, 2007.
- J. T. Betts. Practical Methods for Optimal Control Using Nonlinear Programming. SIAM, 2001.
- S. Boyd, L. Vandenberghe. Convex Optimization. Cambridge University Press, 2004.